Types
for
Information-Flow Control
in
Functional Programming
Languages

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Information Flow Control

Prove absence of *undesired flows* of information in programs.

- {JS , App} does not leak {credit card nr, GPS} to ad provider

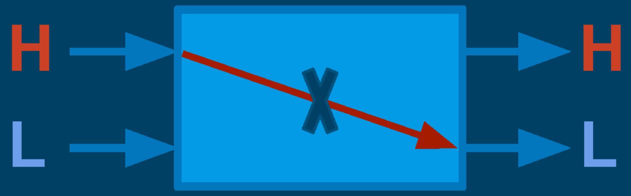
Well developed (40+ years) (but hard)

- OS (seL4), voting system (Civitas), web framework (Swift), email client (JPMail)

Information Flow Control

Noninterference

Property



"H inputs do not interfere with L outputs."

Property of behavior — sets of traces

Many flavors — confidentiality / integrity

Enforced — analysis / transformation

Flows

explicit:

```
low := high;
```

implicit:

```
if (high mod 2 == 1) {
    low := 1;
} else {
    low := 0;
}
```

Type systems for IFC check & disallow this.
Several typed **imperative** IFC languages:
Jif, Paragon, Joana, SPARK, JSFlow

Functional Programming Languages

Different Approaches

How do you track information flows in functional programs?

Two mainstream styles:

Flow Caml vs. lio

Label everything
FG (Fine-Grained)

Label I/O interface
CG (Coarse-)

Two extremes.

Expressive Power?

Main result: Both styles *equally expressive!*

Proof: semantics- and type-preserving translation. (à la Abadi et al. *A Core Calculus of Dependency*).

- FG translates to CG.
- CG translates to FG. **QED**

Today's goal

Teach you IFC for FP (rest is extra)

Outline

FG, CG, translations (touch on that)

- Values have information.
- Deconstructing a value transfers its information to the result.

$e\ e \downarrow v$

Recall: a **function** is a value. application deconstructs it...

FG

Flow Caml
(DCC ← SLam Calculus)

e	::=	$x \mid \lambda x\,.\,e \mid e\ e \mid$	λ- calculus +
		$(e, e) \mid \mathsf{fst}(e) \mid \mathsf{snd}(e) \mid$	tuples +
		$\mathsf{inl}(e) \mid \mathsf{inr}(e) \mid \mathsf{case}(e, x.e, x.e) \mid$	sums +
		$\mathsf{new}\ e \mid !e \mid e := e \mid ()$	effects + unit

τ	::=	A^{ℓ}	types (labeled)
A	::=	$\mathsf{unit} \mid \tau \rightarrow^{\ell} \tau \mid \tau \times \tau \mid \tau + \tau \mid \mathsf{ref}\ \tau$	

labels on
unit and τ × τ
are redundant.

Examples

$\Gamma \vdash_{pc} e : \tau$ judgement

structural label: ℓ in A^{ℓ} (in τ).

$(\text{unit}^{L} + \text{unit}^{L})^{H}$ (think "bool^{H}")

reveals information about
structure of sum (inl or inr) (thus H)

propagation of labels.

$\text{bool}^{H} \wedge \text{bool}^{L}$

reveals information about
either side of $\wedge$ (thus, result type bool^{H})

control label: pc in type judgement.

$e_V : (\text{unit}^{L} + \text{unit}^{L})^{H}$
$e_R : (\text{ref nat}^{L})^{L}$
$e = \text{case}(e_V,\ \_.()\ ,\ \_.e_R := 42\ )$

result always (). but,
e reveals whether e_V is inl or inr, via.
(non-)presence of write on e_R.

FG tracks this *control-flow* information, in pc
(raise pc when typing left/right, by label of e_V).
Lower bound on write effects (in e)
(output must be allowed to learn about pc)

FG

Examples

effect label: ℓ in $(\to^{\ell})$.

$e_V : (\text{unit}^L + \text{unit}^L)^H$
$e = \lambda\, x_F.\ \text{case}(x_V,\ \_.() ,\ \_.(x_F\ ()\ )\ )$

If x_F maps unit^L to unit^L, then e will map such x_F to unit^H (that's OK). But, now consider

$e_R : (\text{ref nat}^L)^L$
$e_F : \lambda\_ .\ (e_R := 42)$

$e\ e_F$ conditionally applies e_F. Thus, $e_R := 42$ in e_F leaks e_V (which is H) to L.

$\Gamma \vdash_{pc} e : \tau$ judgement

FG tracks this control-flow information, in $(\to^{\ell})$.
Lower bound on write effects (in function body)

$e_F : (\text{unit}^L \to^L \text{unit}^L)^L$

FG rejects the above example w/ this type.

How to fix: (i.e. why FG rejected this):

- Make e_V L , or make e_F -stuff H
- Remove the assignment

note: that functions of type $(\tau \to^L \tau)^H$ *can* be constructed, passed around, but never applied, as that would leak its own identity.

Look, a
type system!
moving along

$$\frac{}{\Gamma, x:\tau \vdash_{pc} x:\tau}\ \text{FG-var} \qquad \frac{\Gamma, x:\tau_1 \vdash_{\ell_e} e:\tau_2}{\Gamma \vdash_{pc} \lambda x.e : (\tau_1 \xrightarrow{\ell_e} \tau_2)^{\perp}}\ \text{FG-lam}$$

$$\frac{\Gamma \vdash_{pc} e_1 : (\tau_1 \xrightarrow{\ell_e} \tau_2)^{\ell} \quad \Gamma \vdash_{pc} e_2 : \tau_1 \quad \mathcal{L} \vdash \tau_2 \searrow \ell \quad \mathcal{L} \vdash pc \sqcup \ell \sqsubseteq \ell_e}{\Gamma \vdash_{pc} e_1\ e_2 : \tau_2}\ \text{FG-app}$$

$$\frac{\Gamma \vdash_{pc} e_1 : \tau_1 \quad \Gamma \vdash_{pc} e_2 : \tau_2}{\Gamma \vdash_{pc} (e_1, e_2) : (\tau_1 \times \tau_2)^{\perp}}\ \text{FG-prod} \qquad \frac{\Gamma \vdash_{pc} e : (\tau_1 \times \tau_2)^{\ell} \quad \mathcal{L} \vdash \tau_1 \searrow \ell}{\Gamma \vdash_{pc} \mathsf{fst}(e) : \tau_1}\ \text{FG-fst}$$

$$\frac{\Gamma \vdash_{pc} e : \tau_1}{\Gamma \vdash_{pc} \mathsf{inl}(e) : (\tau_1 + \tau_2)^{\perp}}\ \text{FG-inl}$$

$$\frac{\Gamma \vdash_{pc} e : (\tau_1 + \tau_2)^{\ell} \quad \Gamma, x:\tau_1 \vdash_{pc \sqcup \ell} e_1 : \tau \quad \Gamma, y:\tau_2 \vdash_{pc \sqcup \ell} e_2 : \tau \quad \mathcal{L} \vdash \tau \searrow \ell}{\Gamma \vdash_{pc} \mathsf{case}(e, x.e_1, y.e_2) : \tau}\ \text{FG-case}$$

$$\frac{\Gamma \vdash_{pc'} e : \tau' \quad \mathcal{L} \vdash pc \sqsubseteq pc' \quad \mathcal{L} \vdash \tau' <: \tau}{\Gamma \vdash_{pc} e : \tau}\ \text{FG-sub} \qquad \frac{\Gamma \vdash_{pc} e : \tau \quad \mathcal{L} \vdash \tau \searrow pc}{\Gamma \vdash_{pc} \mathsf{new}\ e : (\mathsf{ref}\ \tau)^{\perp}}\ \text{FG-ref}$$

$$\frac{\Gamma \vdash_{pc} e : (\mathsf{ref}\ \tau)^{\ell} \quad \mathcal{L} \vdash \tau <: \tau' \quad \mathcal{L} \vdash \tau' \searrow \ell}{\Gamma \vdash_{pc} !e : \tau'}\ \text{FG-deref}$$

$$\frac{\Gamma \vdash_{pc} e_1 : (\mathsf{ref}\ \tau)^{\ell} \quad \Gamma \vdash_{pc} e_2 : \tau \quad \mathcal{L} \vdash \tau \searrow (pc \sqcup \ell)}{\Gamma \vdash_{pc} e_1 := e_2 : \mathsf{unit}}\ \text{FG-assign} \qquad \frac{}{\Gamma \vdash_{pc} () : \mathsf{unit}^{\perp}}\ \text{FG-unitI}$$

FG

Types

CG

LIO
(Static fragment of HLIO)

e ::=	x | $\lambda x . e$ | $e\ e$ |	λ- calculus +
	(e, e) | fst(e) | snd(e) |	tuples +
	inl(e) | inr(e) | case(e, $x.e$, $x.e$) |	sums +
	new e | !e | e := e | () |	effects + unit
	lb(e) | unLb(e) | toLb(e) |	labeling +
	ret (e) | bind(e, $x.e$)	label monad

τ ::=	unit | $\tau \rightarrow \tau$ | $\tau \times \tau$ | $\tau + \tau$ | ref ℓ τ |	
	Labeled ℓ τ | ©ℓ_1 ℓ_2 τ	types

Examples

Γ |-- $e : \tau$ judgement

Labeled ℓ τ	like τ^{ℓ} before
ref ℓ τ	stores Labeled ℓ τ (only)
$\mathbb{C}\ \ell_1\ \ell_2\ \tau$	suspended *effectful* computation
input	(computations learns info)
unlabel(e)	(don't learn on read, but on unLabel)
output	(computation releases info)
Label(e)	(or write to ref. that's not strictly needed, though)

ℓ_1 is *pc* ; lower bound on output.
ℓ_2 is *taint* ; upper bound of all unLb

CG only tracks flows through bind.

references: consider the program

e_R = new 0
e_V = label(4)
e_A = x_R := x_V
e = bind(e_R , x_R.bind(ret(e_V) , x_V. e_A)

Consider typing

e_R : $\mathbb{C}$ L L (ref L nat)
e_V : Labeled H nat

Then e does not typecheck; e_A writes (Labeled H nat) to (ref L nat). CG checks this.

CG

Examples

Γ |-- $e : \tau$ judgement

bind: consider instead the program

$e_A = e_{RO}$:= label(4)
e = bind(!e_{RI} , x_I.bind(unlabel(x_I) , $x_\bot$. e_A)

Consider typing

e_{RI} : ref H nat
e_{RO} : ref L nat

CG rejects e w/ these types;
unlabel raises *pc* of e_A to H, making e_A untypeable.

CG tracks this flow through bind.

How to fix (i.e. why CG rejected it): Either

- Make e_{RI} and e_{RO} both H or both L
- Don't unlabel x_I
- Use toLabeled(·) around the part using x_I

That last option added to deal with *label creep*.
(label creep: label monotonely increasing through a bind-chain). Can now do H sub-computation.

Types

Look,
another one!
moving along

(All rules of the simply typed lambda-calculus pertaining to the types b, $\tau \to \tau, \tau \times \tau, \tau + \tau$, and unit are included.)

$$\frac{\Gamma \vdash e_1 : \mathbb{C}\ \ell_1\ \ell_2\ \tau \qquad \Gamma, x:\tau \vdash e_2 : \mathbb{C}\ \ell_3\ \ell_4\ \tau' \qquad \ell \sqsubseteq \ell_1 \qquad \ell \sqsubseteq \ell_3 \qquad \ell_2 \sqsubseteq \ell_3 \qquad \ell_2 \sqsubseteq \ell_4}{\Gamma \vdash \mathsf{bind}(e_1, x.e_2) : \mathbb{C}\ \ell\ \ell_4\ \tau'}\ \text{CG-bind}$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathsf{ret}(e) : \mathbb{C}\ \top\ \bot\ \tau}\ \text{CG-ret} \qquad \frac{\Gamma \vdash e : \tau' \qquad \mathcal{L} \vdash \tau' <: \tau}{\Gamma \vdash e : \tau}\ \text{CG-sub} \qquad \frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathsf{Lb}(e) : \mathsf{Labeled}\ \ell\ \tau}\ \text{CG-label}$$

$$\frac{\Gamma \vdash e : \mathsf{Labeled}\ \ell\ \tau}{\Gamma \vdash \mathsf{unlabel}(e) : \mathbb{C}\ \top\ \ell\ \tau}\ \text{CG-unlabel} \qquad \frac{\Gamma \vdash e : \mathsf{Labeled}\ \ell\ \tau}{\Gamma \vdash \mathsf{new}\ e : \mathbb{C}\ \ell\ \bot\ (\mathsf{ref}\ \ell\ \tau)}\ \text{CG-ref} \qquad \frac{\Gamma \vdash e : \mathsf{ref}\ \ell'\ \tau}{\Gamma \vdash\ !e : \mathbb{C}\ \top\ \bot\ (\mathsf{Labeled}\ \ell'\tau)}\ \text{CG-deref}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{ref}\ \ell\ \tau \qquad \Gamma \vdash e_2 : \mathsf{Labeled}\ \ell\ \tau}{\Gamma \vdash e_1 := e_2 : \mathbb{C}\ \ell\ \bot\ \mathsf{unit}}\ \text{CG-assign} \qquad \frac{\Gamma \vdash e : \mathbb{C}\ \ell\ \ell'\ \tau}{\Gamma \vdash \mathsf{toLabeled}(e) : \mathbb{C}\ \ell\ \bot\ (\mathsf{Labeled}\ \ell'\ \tau)}\ \text{CG-toLabeled}$$

Subtyping judgment: $\boxed{\mathcal{L} \vdash \tau <: \tau'}$

$$\frac{\mathcal{L} \vdash \tau <: \tau' \qquad \mathcal{L} \vdash \ell \sqsubseteq \ell'}{\mathcal{L} \vdash \mathsf{Labeled}\ \ell\ \tau <: \mathsf{Labeled}\ \ell'\ \tau'}\ \text{CGsub-labeled} \qquad \frac{\mathcal{L} \vdash \tau <: \tau' \qquad \mathcal{L} \vdash \ell_1' \sqsubseteq \ell_1 \qquad \mathcal{L} \vdash \ell_2 \sqsubseteq \ell_2'}{\mathcal{L} \vdash \mathbb{C}\ \ell_1\ \ell_2\ \tau <: \mathbb{C}\ \ell_1'\ \ell_2'\ \tau'}\ \text{CGsub-monad}$$

Equivalence

FG *seems* more expressive;
it tracks flows at finer granularity.

But what if you structure code as *many tiny computations* in CG?

Theorem: e well-typed in CG translates to $[\![e]\!]$ well-typed in FG w/ *same behavior*, and vice versa

Proof: semantics- and type-preserving translation (à la DCC)

(sketch follows)

Equivalence

CG → FG

$\Gamma \mid\!\!-\!\!- \; e : \tau \;\sim\!> [\![\Gamma]\!] \mid\!\!-\!\!- \; [\![e]\!] : [\![\tau]\!]$?

Equivalence

CG → FG

Γ |-- $e : \tau$ ~> $[\![\Gamma]\!]$ $|\text{--}_{\top}$ $[\![e]\!] : [\![\tau]\!]$

All CG-computation is suspended

Equivalence

CG → FG $\Gamma \vdash e : \tau \rightsquigarrow [\![\Gamma]\!] \vdash_{\top} [\![e]\!] : [\![\tau]\!]$

Types

First four trivial (CG unlabeled)

Labeled ℓ τ Label [[τ]] with ℓ, how? Already labeled.
Coding trick.
We ensure expr. always eval to inl.

ref ℓ τ Label [[τ]] with ℓ, same trick ^

$\mathbb{C}$ ℓ_1 ℓ_2 τ $\mathbb{C}$ is suspended. so, map it to a *thunk*.
ℓ_1 is bound on writes. Put that on →.
ℓ_2 is value label. Label [[τ]] with ℓ ^

$$
\begin{aligned}
[\![\mathsf{b}]\!] &= \mathsf{b}^{\perp} \\
[\![\tau_1 \to \tau_2]\!] &= ([\![\tau_1]\!] \xrightarrow{\top} [\![\tau_2]\!])^{\perp} \\
[\![\tau_1 \times \tau_2]\!] &= ([\![\tau_1]\!] \times [\![\tau_2]\!])^{\perp} \\
[\![\tau_1 + \tau_2]\!] &= ([\![\tau_1]\!] + [\![\tau_2]\!])^{\perp} \\
[\![\mathsf{ref}\ \ell\ \tau]\!] &= (\mathsf{ref}\ ([\![\tau]\!] + \mathsf{unit})^{\ell})^{\perp} \\
[\![\mathbb{C}\ \ell_1\ \ell_2\ \tau]\!] &= (\mathsf{unit} \xrightarrow{\ell_1} ([\![\tau]\!] + \mathsf{unit})^{\ell_2})^{\perp} \\
[\![\mathsf{Labeled}\ \ell\ \tau]\!] &= ([\![\tau]\!] + \mathsf{unit})^{\ell}
\end{aligned}
$$

Equivalence

CG → FG $\Gamma \vdash e : \tau \rightsquigarrow [\![\Gamma]\!] \vdash_{\top} [\![e]\!] : [\![\tau]\!]$

Expressions

Translates a well-typed CG-expression (i.e. proof of $\Gamma \vdash e : \tau$),
into a well-typed FG-expression (i.e. proof of $[\![\Gamma]\!] \vdash_{\top} [\![e]\!] : [\![\tau]\!]$).

$$\frac{\Gamma \vdash e_1 : \mathsf{CG}\ \ell_i\ \ell\ \tau \qquad \Gamma, x : \tau \vdash e_2 : \mathsf{CG}\ \ell\ \ell_o\ \tau'}{\Gamma \vdash \mathsf{bind}(e_1, x.e_2) : \mathsf{CG}\ \ell_i\ \ell_o\ \tau'} \rightsquigarrow$$

$$\frac{[\![\Gamma]\!] \vdash_{\top} e_1' : (\mathsf{unit} \xrightarrow{\ell_i} ([\![\tau]\!] + \mathsf{unit})^{\ell})^{\perp} \qquad [\![\Gamma]\!], x : [\![\tau]\!] \vdash_{\top} e_2' : (\mathsf{unit} \xrightarrow{\ell} ([\![\tau']\!] + \mathsf{unit})^{\ell_o})^{\perp}}{[\![\Gamma]\!] \vdash_{\top} \lambda\_.\mathsf{case}(e_1'(), x.e_2'(), y.\mathsf{inr}()) : (\mathsf{unit} \xrightarrow{\ell_i} ([\![\tau']\!] + \mathsf{unit})^{\ell_o})^{\perp}}$$

Equivalence

FG → CG

$\Gamma \vdash_{pc} e : \tau \rightsquigarrow (\!|\Gamma|\!) \vdash (\!|e|\!) : \quad (\!|\tau|\!) \quad ?$

Equivalence

FG → CG

$$\Gamma \vdash_{pc} e : \tau \rightsquigarrow (\!|\Gamma|\!) \vdash (\!|e|\!) : \mathbb{C}\; pc \perp (\!|\tau|\!)$$

Track FG-pc
With CG-monad

Equivalence

FG → CG $\Gamma \vdash_{pc} e : \tau \rightsquigarrow (\!|\Gamma|\!) \vdash (\!|e|\!) : \mathbb{C}\ pc\ \bot\ (\!|\tau|\!)$

Types

All cases trivial (use Labeled), save for one.

$\tau_1 \xrightarrow{\ell_e} \tau_2$ Function that returns a monadic computation.
monad confines I/O of original f.
ℓ_e is lower bound on writes (like pc).
Put that in $\mathbb{C}$.

$(\!|e|\!)$ boxes everything with toLabeled, hence the $\bot$.

$$
\begin{array}{lcll}
(\!|\mathsf{b}|\!) & = & \mathsf{b} & \\
(\!|\mathsf{unit}|\!) & = & \mathsf{unit} & \\
(\!|\tau_1 \xrightarrow{\ell_e} \tau_2|\!) & = & (\!|\tau_1|\!) \to \mathbb{C}\ \ell_e\ \bot\ (\!|\tau_2|\!) & \\
(\!|\tau_1 \times \tau_2|\!) & = & (\!|\tau_1|\!) \times (\!|\tau_2|\!) & \\
(\!|\tau_1 + \tau_2|\!) & = & (\!|\tau_1|\!) + (\!|\tau_2|\!) & \\
(\!|\mathsf{ref}\ \tau|\!) & = & \mathsf{ref}\ \ell\ (\!|\mathsf{A}|\!) & \text{when } \tau = \mathsf{A}^{\ell} \\
(\!|\mathsf{A}^{\ell}|\!) & = & \mathsf{Labeled}\ \ell\ (\!|\mathsf{A}|\!) &
\end{array}
$$

Equivalence

FG → CG

$$\Gamma \mid\!\!\text{--}_{pc}\ e : \tau \rightsquigarrow (\!|\Gamma|\!) \mid\!\!\text{--}\ (\!|e|\!) : \mathbb{C}\ pc \perp (\!|\tau|\!)$$

Expressions

$$\frac{\Gamma \vdash_{pc} e_{\mathtt{F}} : (\tau_{\mathtt{I}} \xrightarrow{\ell_e} \tau_{\mathtt{O}})^{\ell} \qquad \Gamma \vdash_{pc} e_{\mathtt{I}} : \tau_{\mathtt{I}} \qquad \mathcal{L} \vdash \tau_{\mathtt{O}} \searrow \ell \qquad \mathcal{L} \vdash pc \sqcup \ell \sqsubseteq \ell_e}{\Gamma \vdash_{pc} e_{\mathtt{F}}\ e_{\mathtt{I}} : \tau_{\mathtt{O}}}\ \text{FG-app} \rightsquigarrow$$

$$\frac{?}{(\!|\Gamma|\!) \vdash (\!|e_{\mathtt{F}}\ e_{\mathtt{I}}|\!) : \mathsf{CG}\ \ell_e \perp (\!|\tau_{\mathtt{O}}|\!)}\ \text{CG-toLabeled}$$

Huge derivation

$$\begin{aligned}
(\!|e_{\mathtt{F}}|\!) &: \mathsf{CG}\ pc \perp (\!|(\tau_{\mathtt{I}} \xrightarrow{\ell_e} \tau_{\mathtt{O}})^{\ell}|\!) \\
&\ \mathsf{CG}\ pc \perp (\ \mathsf{Labeled}\ \ell\ ((\!|\tau_{\mathtt{I}}|\!) \to \mathsf{CG}\ \ell_e \perp (\!|\tau_{\mathtt{O}}|\!))\ ) \\
(\!|e_{\mathtt{I}}|\!) &: \mathsf{CG}\ pc \perp (\!|\tau_{\mathtt{I}}|\!) \\
(\!|e_{\mathtt{F}}\ e_{\mathtt{I}}|\!) &: \mathsf{CG}\ \ell_e \perp (\!|\tau_{\mathtt{O}}|\!)
\end{aligned}$$

$$(\!|e_{\mathtt{F}}\ e_{\mathtt{I}}|\!) = \mathsf{toLabeled}(\ \mathsf{bind}(\ \mathsf{bind}(\ (\!|e_{\mathtt{F}}|\!)\ ,\ x_{\mathtt{LF}}.\mathsf{bind}(\ (\!|e_{\mathtt{I}}|\!)\ ,\ x_{\mathtt{I}}.\mathsf{bind}(\ \mathsf{unlabel}(x_{\mathtt{LF}})\ ,\ x_{\mathtt{F}}.x_{\mathtt{F}}\ x_{\mathtt{I}})))\ ,\ x_{\mathtt{LO}}.\mathsf{unlabel}(x_{\mathtt{LO}}))\ )$$

CG↔FG, Semantics

Last (crucial) step:

- Type Soundness
- Semantics-Preservation

Co-author (Vineet Rajani) did this in a follow-up paper!

- Step-indexed Kripke *logical relation*
  (based on a model of types
  in incremental computational complexity).

Summary

- IFC in FP crash-course
- FG and CG different granularity
- FG and CG equally expressive

  Proof: semantics- and type-preserving translation.

  logical relations.
- Suffices to use CG

Acknowledgements

Willard Rafnsson

Iulia Bastys

Vineet Rajani

Deepak Garg

SIGLOG News 2017

Type Systems for Information Flow Control: The Question of Granularity

CSF 2018

Types for Information Flow Control: Labeling Granularity and Semantic Models

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Appendix

FG

Types

Typing judgment: $\boxed{\Gamma \vdash_{pc} e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash_{pc} x : \tau}\ \text{FG-var} \qquad \frac{\Gamma, x : \tau_1 \vdash_{\ell_e} e : \tau_2}{\Gamma \vdash_{pc} \lambda x.e : (\tau_1 \xrightarrow{\ell_e} \tau_2)^{\perp}}\ \text{FG-lam}$$

$$\frac{\Gamma \vdash_{pc} e_1 : (\tau_1 \xrightarrow{\ell_e} \tau_2)^{\ell} \qquad \Gamma \vdash_{pc} e_2 : \tau_1 \qquad \mathcal{L} \vdash \tau_2 \searrow \ell \qquad \mathcal{L} \vdash pc \sqcup \ell \sqsubseteq \ell_e}{\Gamma \vdash_{pc} e_1\ e_2 : \tau_2}\ \text{FG-app}$$

$$\frac{\Gamma \vdash_{pc} e_1 : \tau_1 \qquad \Gamma \vdash_{pc} e_2 : \tau_2}{\Gamma \vdash_{pc} (e_1, e_2) : (\tau_1 \times \tau_2)^{\perp}}\ \text{FG-prod} \qquad \frac{\Gamma \vdash_{pc} e : (\tau_1 \times \tau_2)^{\ell} \qquad \mathcal{L} \vdash \tau_1 \searrow \ell}{\Gamma \vdash_{pc} \mathsf{fst}(e) : \tau_1}\ \text{FG-fst}$$

$$\frac{\Gamma \vdash_{pc} e : \tau_1}{\Gamma \vdash_{pc} \mathsf{inl}(e) : (\tau_1 + \tau_2)^{\perp}}\ \text{FG-inl}$$

$$\frac{\Gamma \vdash_{pc} e : (\tau_1 + \tau_2)^{\ell} \qquad \Gamma, x : \tau_1 \vdash_{pc \sqcup \ell} e_1 : \tau \qquad \Gamma, y : \tau_2 \vdash_{pc \sqcup \ell} e_2 : \tau \qquad \mathcal{L} \vdash \tau \searrow \ell}{\Gamma \vdash_{pc} \mathsf{case}(e, x.e_1, y.e_2) : \tau}\ \text{FG-case}$$

$$\frac{\Gamma \vdash_{pc'} e : \tau' \qquad \mathcal{L} \vdash pc \sqsubseteq pc' \qquad \mathcal{L} \vdash \tau' <: \tau}{\Gamma \vdash_{pc} e : \tau}\ \text{FG-sub} \qquad \frac{\Gamma \vdash_{pc} e : \tau \qquad \mathcal{L} \vdash \tau \searrow pc}{\Gamma \vdash_{pc} \mathsf{new}\ e : (\mathsf{ref}\ \tau)^{\perp}}\ \text{FG-ref}$$

$$\frac{\Gamma \vdash_{pc} e : (\mathsf{ref}\ \tau)^{\ell} \qquad \mathcal{L} \vdash \tau <: \tau' \qquad \mathcal{L} \vdash \tau' \searrow \ell}{\Gamma \vdash_{pc} !e : \tau'}\ \text{FG-deref}$$

$$\frac{\Gamma \vdash_{pc} e_1 : (\mathsf{ref}\ \tau)^{\ell} \qquad \Gamma \vdash_{pc} e_2 : \tau \qquad \mathcal{L} \vdash \tau \searrow (pc \sqcup \ell)}{\Gamma \vdash_{pc} e_1 := e_2 : \mathsf{unit}}\ \text{FG-assign} \qquad \frac{}{\Gamma \vdash_{pc} () : \mathsf{unit}^{\perp}}\ \text{FG-unitI}$$

FG

Types

Typing judgment: $\boxed{\Gamma \vdash_{pc} e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash_{pc} x : \tau} \text{ FG-var}$$

$$\frac{}{\Gamma \vdash_{pc} () : \mathsf{unit}^{\perp}} \text{ FG-unitI}$$

FG

Types

Always safe to
- **up-classify** information
- **weaken** the guarantee (*)

Rule FG-sub: ↓
- raise τ
- lower *pc* (*)

Raise τ means
- raise structural ℓ
- lower τ_I and ℓ in $\tau_I \to^{\ell} \tau_O$ (*)

Typing judgment: $\boxed{\Gamma \vdash_{pc} e : \tau}$

$$\frac{\Gamma \vdash_{pc'} e : \tau' \qquad \mathcal{L} \vdash pc \sqsubseteq pc' \qquad \mathcal{L} \vdash \tau' <: \tau}{\Gamma \vdash_{pc} e : \tau} \text{ FG-sub}$$

FG

Types

Always safe to
- **up-classify** &
- **weaken**

$A^{\ell} \searrow \ell'$ just means $\ell \sqsubseteq \ell'$

(prevent leaking context into result)

Typing judgment: $\boxed{\Gamma \vdash_{pc} e : \tau}$

$$\frac{\Gamma \vdash_{pc} e : \tau_1}{\Gamma \vdash_{pc} \mathsf{inl}(e) : (\tau_1 + \tau_2)^{\perp}} \text{ FG-inl}$$

$$\frac{\Gamma \vdash_{pc} e : (\tau_1 + \tau_2)^{\ell} \quad \Gamma, x : \tau_1 \vdash_{pc \sqcup \ell} e_1 : \tau \quad \Gamma, y : \tau_2 \vdash_{pc \sqcup \ell} e_2 : \tau \quad \mathcal{L} \vdash \tau \searrow \ell}{\Gamma \vdash_{pc} \mathsf{case}(e, x.e_1, y.e_2) : \tau} \text{ FG-case}$$

FG

Types

Always safe to
- **up-classify** &
- **weaken**

$A^{\ell} \searrow \ell' \Leftrightarrow \ell \sqsubseteq \ell'$

FG-lam, why ■ :
- no evaluation; only checking that ℓ_e bounds effects below.
- FG-case , FG-app put pc into structural info.

Typing judgment: $\boxed{\Gamma \vdash_{pc} e : \tau}$

$$\frac{\Gamma, x : \tau_1 \vdash_{\ell_e} e : \tau_2}{\Gamma \vdash_{pc} \lambda x.e : (\tau_1 \xrightarrow{\ell_e} \tau_2)^{\perp}} \text{ FG-lam}$$

$$\frac{\Gamma \vdash_{pc} e_1 : (\tau_1 \xrightarrow{\ell_e} \tau_2)^{\ell} \quad \Gamma \vdash_{pc} e_2 : \tau_1 \quad \mathcal{L} \vdash \tau_2 \searrow \ell \quad \mathcal{L} \vdash pc \sqcup \ell \sqsubseteq \ell_e}{\Gamma \vdash_{pc} e_1\ e_2 : \tau_2} \text{ FG-app}$$

CG

Types

Typing judgment: $\boxed{\Gamma \vdash e : \tau}$

(All rules of the simply typed lambda-calculus pertaining to the types $\mathsf{b}, \tau \to \tau, \tau \times \tau, \tau + \tau$, and unit are included.)

$$\frac{\Gamma \vdash e_1 : \mathbb{C}\; \ell_1\; \ell_2\; \tau \qquad \Gamma, x : \tau \vdash e_2 : \mathbb{C}\; \ell_3\; \ell_4\; \tau' \qquad \ell \sqsubseteq \ell_1 \qquad \ell \sqsubseteq \ell_3 \qquad \ell_2 \sqsubseteq \ell_3 \qquad \ell_2 \sqsubseteq \ell_4}{\Gamma \vdash \mathsf{bind}(e_1, x.e_2) : \mathbb{C}\; \ell\; \ell_4\; \tau'}\;\text{CG-bind}$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathsf{ret}(e) : \mathbb{C}\; \top\; \bot\; \tau}\;\text{CG-ret} \qquad \frac{\Gamma \vdash e : \tau' \qquad \mathcal{L} \vdash \tau' <: \tau}{\Gamma \vdash e : \tau}\;\text{CG-sub} \qquad \frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathsf{Lb}(e) : \mathsf{Labeled}\; \ell\; \tau}\;\text{CG-label}$$

$$\frac{\Gamma \vdash e : \mathsf{Labeled}\; \ell\; \tau}{\Gamma \vdash \mathsf{unlabel}(e) : \mathbb{C}\; \top\; \ell\; \tau}\;\text{CG-unlabel} \qquad \frac{\Gamma \vdash e : \mathsf{Labeled}\; \ell\; \tau}{\Gamma \vdash \mathsf{new}\; e : \mathbb{C}\; \ell\; \bot\; (\mathsf{ref}\; \ell\; \tau)}\;\text{CG-ref} \qquad \frac{\Gamma \vdash e : \mathsf{ref}\; \ell'\; \tau}{\Gamma \vdash !e : \mathbb{C}\; \top\; \bot\; (\mathsf{Labeled}\; \ell'\tau)}\;\text{CG-deref}$$

$$\frac{\Gamma \vdash e_1 : \mathsf{ref}\; \ell\; \tau \qquad \Gamma \vdash e_2 : \mathsf{Labeled}\; \ell\; \tau}{\Gamma \vdash e_1 := e_2 : \mathbb{C}\; \ell\; \bot\; \mathsf{unit}}\;\text{CG-assign} \qquad \frac{\Gamma \vdash e : \mathbb{C}\; \ell\; \ell'\; \tau}{\Gamma \vdash \mathsf{toLabeled}(e) : \mathbb{C}\; \ell\; \bot\; (\mathsf{Labeled}\; \ell'\; \tau)}\;\text{CG-toLabeled}$$

Subtyping judgment: $\boxed{\mathcal{L} \vdash \tau <: \tau'}$

$$\frac{\mathcal{L} \vdash \tau <: \tau' \qquad \mathcal{L} \vdash \ell \sqsubseteq \ell'}{\mathcal{L} \vdash \mathsf{Labeled}\; \ell\; \tau <: \mathsf{Labeled}\; \ell'\; \tau'}\;\text{CGsub-labeled} \qquad \frac{\mathcal{L} \vdash \tau <: \tau' \qquad \mathcal{L} \vdash \ell_1' \sqsubseteq \ell_1 \qquad \mathcal{L} \vdash \ell_2 \sqsubseteq \ell_2'}{\mathcal{L} \vdash \mathbb{C}\; \ell_1\; \ell_2\; \tau <: \mathbb{C}\; \ell_1'\; \ell_2'\; \tau'}\;\text{CGsub-monad}$$

CG

Types

Typing judgment: $\boxed{\Gamma \vdash e : \tau}$

$$\frac{\Gamma \vdash e_1 : \mathbb{C}\;\ell_1\;\ell_2\;\tau \qquad \Gamma, x : \tau \vdash e_2 : \mathbb{C}\;\ell_3\;\ell_4\;\tau' \qquad \ell \sqsubseteq \ell_1 \qquad \ell \sqsubseteq \ell_3 \qquad \ell_2 \sqsubseteq \ell_3 \qquad \ell_2 \sqsubseteq \ell_4}{\Gamma \vdash \mathsf{bind}(e_1, x.e_2) : \mathbb{C}\;\ell\;\ell_4\;\tau'}\ \text{CG-bind}$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathsf{ret}(e) : \mathbb{C}\;\top\;\bot\;\tau}\ \text{CG-ret}$$

CG

Types

Typing judgment: $\boxed{\Gamma \vdash e : \tau}$

$$\frac{\Gamma \vdash e : \tau' \qquad \mathcal{L} \vdash \tau' <: \tau}{\Gamma \vdash e : \tau} \text{ CG-sub}$$

Subtyping judgment: $\boxed{\mathcal{L} \vdash \tau <: \tau'}$

$$\frac{\mathcal{L} \vdash \tau <: \tau' \qquad \mathcal{L} \vdash \ell \sqsubseteq \ell'}{\mathcal{L} \vdash \mathsf{Labeled}\ \ell\ \tau <: \mathsf{Labeled}\ \ell'\ \tau'} \text{ CGsub-labeled}$$

$$\frac{\mathcal{L} \vdash \tau <: \tau' \qquad \mathcal{L} \vdash \ell_1' \sqsubseteq \ell_1 \qquad \mathcal{L} \vdash \ell_2 \sqsubseteq \ell_2'}{\mathcal{L} \vdash \mathbb{C}\ \ell_1\ \ell_2\ \tau <: \mathbb{C}\ \ell_1'\ \ell_2'\ \tau'} \text{ CGsub-monad}$$

up-classify
& **weaken**

Types

Typing judgment: $\boxed{\Gamma \vdash e : \tau}$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathsf{Lb}(e) : \mathsf{Labeled}\ \ell\ \tau}\ \text{CG-label}$$

$$\frac{\Gamma \vdash e : \mathsf{Labeled}\ \ell\ \tau}{\Gamma \vdash \mathsf{unlabel}(e) : \mathbb{C}\ \top\ \ell\ \tau}\ \text{CG-unlabel}$$

$$\frac{\Gamma \vdash e : \mathbb{C}\ \ell\ \ell'\ \tau}{\Gamma \vdash \mathsf{toLabeled}(e) : \mathbb{C}\ \ell\ \bot\ (\mathsf{Labeled}\ \ell'\ \tau)}\ \text{CG-toLabeled}$$

Types

Typing judgment: $\boxed{\Gamma \vdash e : \tau}$

$$\frac{\Gamma \vdash e : \textsf{Labeled}\ \ell\ \tau}{\Gamma \vdash \textsf{new}\ e : \mathbb{C}\ \ell\ \bot\ (\textsf{ref}\ \ell\ \tau)}\ \text{CG-ref}$$

$$\frac{\Gamma \vdash e : \textsf{ref}\ \ell'\ \tau}{\Gamma \vdash !e : \mathbb{C}\ \top\ \bot\ (\textsf{Labeled}\ \ell'\tau)}\ \text{CG-deref}$$

$$\frac{\Gamma \vdash e_1 : \textsf{ref}\ \ell\ \tau \qquad \Gamma \vdash e_2 : \textsf{Labeled}\ \ell\ \tau}{\Gamma \vdash e_1 := e_2 : \mathbb{C}\ \ell\ \bot\ \textsf{unit}}\ \text{CG-assign}$$