Type Theory in the Software Analysis Workbench

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Type Theory Based Tools
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Goals

• Describe an existing system: the Software Analysis Workbench (SAW)
  ▶ Tool for constructing functional models of imperative programs
  ▶ Shared intermediate language nominally based on type theory
  ▶ Heavy use of automated reasoning

• Solicit input on future directions
  ▶ Type theory has much more promise
  ▶ How best to use it?
What is SAW?

• SAW = Software Analysis Workbench
  ▶ Software: many languages
  ▶ Analysis: many types of analysis, focused on functionality
  ▶ Workbench: flexible interface, supporting many goals

• Intended as a flexible tool for software analysis

• What separates it from other systems?
  ▶ One view: compiler :: imperative code → functional code
  ▶ Captures all functional behavior, simplifying later if necessary
  ▶ Uses efficient internal representations tuned to equivalence checking
  ▶ Strong bit vector reasoning support
  ▶ Focus on practicality over novelty

• Open source (BSD3) and available now
A single, high-level specification for (cryptographic) algorithms

• Cryptol goals
  ▶ Appropriate for cryptography
  ▶ Natural
  ▶ Concise
  ▶ Similar to existing notation
  ▶ Appropriate for execution and verification

• Language features
  ▶ Statically-typed functional language
  ▶ Sized bit vectors (type level naturals)
  ▶ Stream comprehensions (stream diagrams)
Functions and sequences are key notions
Both can be recursive
To compute the sequence of all natural numbers

\[
\text{nats} = [0] \# [ n + 1 \mid n \leftarrow \text{nats} ]
\]
Relationship Between Cryptol and SAW

- Cryptol is essentially the expression language of SAWScript
- Built-in support for Cryptol syntax
  - Translated automatically into Term objects with `{{ ... }}`
- Emerged as an evolution of Cryptol REPL commands
  - Generalizes more constrained `prove` and `sat`
  - More complete language
  - Beyond automated proofs
- Supports proofs purely on Cryptol
- Allows proofs comparing Cryptol to real-world implementations
• Proofs work on Term objects that have result type Bit
• Includes any Cryptol function with result type Bit, as well as terms coming from other sources
• The best-performing prover depends heavily on the problem

```
sawscript> let {{ p (x: [4096]) = x+x+x+x == x*4 }}
sawscript> time (prove abc {{ p }})
```
• Proofs work on `Term` objects that have result type `Bit`
• Includes any Cryptol function with result type `Bit`, as well as terms coming from other sources
• The best-performing prover depends heavily on the problem

```
sawscript> let {{ p (x:[4096]) = x+x+x+x == x*4 }}
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Time: 3.433s
Valid
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sawscript> let {{ p (x:[4096]) = x+x+x+x == x*4 }}
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Time: 3.433s
Valid
sawscript> time (prove z3 {{ p }})
• Proofs work on Term objects that have result type Bit
• Includes any Cryptol function with result type Bit, as well as terms coming from other sources
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```sawscript
sawscript> let {{ p (x:[4096]) = x+x+x+x == x*4 }}
sawscript> time (prove abc {{ p }})
Time: 3.433s
Valid
sawscript> time (prove z3 {{ p }})
Time: 0.006s
Valid
```
import "DES.cry";
let {{ enc = DES.encrypt }};
let {{ dec = DES.decrypt }};
dec_enc <- time (prove abc {{ \k m -> dec k (enc k m) == m }});
enc_dec <- time (prove abc {{ \k m -> enc k (dec k m) == m }});
let ss = simpset [dec_enc, enc_dec];
let {{
    enc3 k1 k2 k3 msg = enc k3 (dec k2 (enc k1 msg))
    dec3 k1 k2 k3 msg = dec k1 (enc k2 (dec k3 msg))
    dec3_enc3 k1 k2 k3 msg = dec3 k1 k2 k3 (enc3 k1 k2 k3 msg) == msg
}};
time (prove do { simplify ss; abc; } {{ dec3_enc3 }});
import "DES.cry";
let {{ enc = DES.encrypt }};
let {{ dec = DES.decrypt }};
dec_enc <- time (prove abc {{ \k m -> dec k (enc k m) == m }});
enc_dec <- time (prove abc {{ \k m -> enc k (dec k m) == m }});
let ss = simpset [dec_enc, enc_dec];
let {{
    enc3 k1 k2 k3 msg = enc k3 (dec k2 (enc k1 msg))
    dec3 k1 k2 k3 msg = dec k1 (enc k2 (dec k3 msg))
    dec3_enc3 k1 k2 k3 msg = dec3 k1 k2 k3 (enc3 k1 k2 k3 msg) == msg
}};
time (prove do { simplify ss; abc; } {{ dec3_enc3 }});
More Complex Proof: 3DES

```ml
import "DES.cry";
let {{ enc = DES.encrypt }};
let {{ dec = DES.decrypt }};
dec_enc <- time (prove abc {{ \( k, m \) -> dec k (enc k m) == m }});
enc_dec <- time (prove abc {{ \( k, m \) -> enc k (dec k m) == m }});
let ss = simpset [dec_enc, enc_dec];
let {{
  enc3 k1 k2 k3 msg = enc k3 (dec k2 (enc k1 msg))
  dec3 k1 k2 k3 msg = dec k1 (enc k2 (dec k3 msg))
  dec3_enc3 k1 k2 k3 msg = dec3 k1 k2 k3 (enc3 k1 k2 k3 msg) == msg
}};
time (prove do { simplify ss; abc; } {{ dec3_enc3 }});
```

Valid
Time: 4.694s
Valid
Time: 4.718s
Valid
Time: 0.003s
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import "DES.cry";
let {{ enc = DES.encrypt }};
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let ss = simpset [dec_enc, enc_dec];
let {{
  enc3 k1 k2 k3 msg = enc k3 (dec k2 (enc k1 msg))
  dec3 k1 k2 k3 msg = dec k1 (enc k2 (dec k3 msg))
  dec3_enc3 k1 k2 k3 msg = dec3 k1 k2 k3 (enc3 k1 k2 k3 msg) == msg
}};
time (prove do { simplify ss; abc; } {{ dec3_enc3 }});
```

```
Valid
Time: 4.694s
Valid
Time: 4.718s
Valid
Time: 0.003s
```
static int ffs_ref(int word) {
    if (word == 0) return 0;
    for (int cnt = 0, i = 0; cnt < 32; cnt++)
        if (((1 << i++) & word) != 0) return i;
    return 0;
}

static int ffs_imp(int i) {
    byte n = 1;
    if (((i & 0xffff) == 0) { n += 16; i >>= 16; } 
    if (((i & 0x00ff) == 0) { n += 8; i >>= 8; } 
    if (((i & 0x000f) == 0) { n += 4; i >>= 4; } 
    if (((i & 0x0003) == 0) { n += 2; i >>= 2; } 
    if (i != 0) { return (n+((i+1) & 0x01)); } else { return 0; } 
}

ffs_cls <- java_load_class "FFS";
ffs_ref <- java_extract ffs_cls "ffs_ref" java_pure;
ffs_imp <- java_extract ffs_cls "ffs_imp" java_pure;
prove abc {{ \x -> ffs_ref x == ffs_imp x }}; // Valid: 0.014s
Case Study: AES

- Proved correctness of many implementations using SAW
- Proof is automated but slow: 5m – 2.5h
- Script to prove OpenSSL C implementation in place
  - Likely to be merged into official OpenSSL source tree
  - AES-128 and AES-256, encryption and decryption
  - Slowest equivalence checking (at least 1h for each proof)

- ~1300 C LOC
- ~230 spec LOC
- ~5 script lines per proof (all plumbing)
Case Study: ECDSA

- Elliptic Curve Digital Signature Algorithm (ECDSA)
- In-house Java code, tuned for speed and verifiability
  - Available with SAW distribution
- ~2400 Java LOC
- ~1600 spec LOC
- ~1500 proof script LOC (largely plumbing)
- Proof completes in < 5m
Case Study: HMAC

- Amazon TLS implementation
- Code from official s2n repository
- ~ 15 (top-level) spec LOC (monolithic function)
- ~ 300 C LOC (iterative code)
- ~ 400 script LOC (all plumbing)
- Proofs for various fixed message sizes
  - <1m per proof
Constructing Models with Symbolic Execution

- Imperative $\rightarrow$ functional via symbolic execution
- For straight-line code, symbolic value of any variable at end is a pure function of symbolic inputs
- Model memory ephemerally
- For branches, **merge** symbolic states at post-dominators
  - A nested application of the if-then-else function
- **Unroll** loops
  - So they’re just a case of sequential branching
  - Can terminate more frequently by SAT-checking branch conditions
- Have also experimented with using **fixpoint combinator**
The SAWCore Language

- Dependently typed core calculus
- Takes some inspiration from CiC, some from MLTT
  - More on specifics, future later
- Represented efficiently with hash-consed DAGs
- Large number of primitives
  - Covering, e.g., the SMT `QF_AUFBV` theory
  - Even though these can be (and have been!) defined in SAWCore, too
- Two type checkers
  - One from surface syntax to explicitly type terms
  - One on explicitly typed terms (incomplete)
  - No guarantee that they agree!
• As a type theory, two notions of proof in SAWCore
  ➤ Showing inhabitant of equality type
  ➤ Showing a Boolean term equivalent to True

• Proofs can be performed by SAT and SMT solvers
  ➤ Several tactics for transformation in advance
  ➤ Solvers use classical logic!

• Hand-constructed proof objects are more powerful
  ➤ But no tactics at this level

• Terms of type \text{Eq } a \ b are theorems
• Terms of structure \text{a == b} can be theorems
  ➤ If shown valid by external prover
Rewriting the main proof tactic available

- Both `Eq a b` and `a == b` can be used as rules
  - The latter normally proved before use, but optional
  - Function definitions are collection of rewrite rules

- Symbolic execution can be thought of as an instance of rewriting

- Some limitations:
  - No conditional rewriting (so far)
  - No auto-simplification for associativity, commutativity, etc.

- Other interactive provers are more flexible
  - Though in some cases less efficient (we routinely process multi-GB terms)
  - And not as integrated with automated provers or model extractors
Open Question: Semantics of SAWCore

- Currently: a somewhat unsound, ad-hoc bag of features
- Ideally: choose an existing, well-studied core calculus and implement it faithfully
- Maybe adapt to semantics of Lean?
  - Lean could be directly linked in
  - Haskell bindings to core API already exist
  - Core language is simple
  - Any interactive proof could use Lean tactics
- Coq export for definitions would also be valuable
  - Proofs would probably be prover-specific, though
Open Question: Representing Non-Termination

- SAW’s main goal: representing program semantics
- Many real programs don’t terminate
  - Or at least are hard to prove to be terminating
- What’s the most effective way to represent them?
- Various possibilities, none ideal
  - Distinguish between type and non-type terms at a sort level, a la Zombie (complex)
  - Use co-inductive reasoning (but induction is more straightforward when possible)
  - Use deep embeddings with a flexible interpreter (slow!)
  - Require variants (simple, but more user burden)
• Some interactive proofs already possible
• Mostly: unconditional rewriting followed by automated tools
• Limited to a single proof goal
  ▶ So case splitting is out
  ▶ Induction even farther away
• Considering the possibility for multiple goals
• Also considering integration with existing interactive provers
  ▶ Lean is a prime candidate
Open Question: Proofs About Complex Memory Models

- Currently, memory “erased” from denotations
- Very efficient and powerful when it works
- Limits the class of programs we can handle
- Explicit memory objects in denotations would help
- How to best represent them?
  - SMT array theory probably too impoverished
  - Maybe a different “primitive” type?
  - Something encoded directly in the logic?
SAW: efficient proofs about imperative programs via translation to functional programs + SAT/SMT

- Practical system, used to verify real-world code, such as:
  - AES from OpenSSL
  - HMAC, DRBG from s2n
  - ECDSA from Galois
  - Portions of several Curve25519 implementations

- Use of types gives structuring principles, helps detect mistakes

- Type theory provides power and flexibility
  - and an explicit form okay, since terms are automatically constructed

- But what possibilities have we yet to take advantage of?
Contributors

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• Cryptol
  ▶ Web: http://cryptol.net
  ▶ GitHub: https://github.com/GaloisInc/cryptol

• Software Analysis Workbench
  ▶ Web: https://saw.galois.com
  ▶ GitHub: https://github.com/GaloisInc/saw-script

• HMAC verification blog post:
  ▶ https://galois.com/blog/2016/09/verifying-s2n-hmac-with-saw/