Fast Elaboration for Dependent Type Theories

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Motivation, overview

Performance issues in current proof assistants.
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Even greater performance demands on future proof assistants.
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Current goals:
- Considering elaboration from ground-up, with performance as priority.
- Benchmarking a prototype against Coq and Agda.
Elaboration

Computing (explicit, well-typed) core from (implicit, incomplete) source language. Includes type checking, unification, desugaring, tactics, etc.

Minimal example for filling holes:

\[
\text{id} : (A : \text{Set}) \rightarrow A \rightarrow A \\
\text{id} \ A \ x = x
\]

\[
\text{id}' : (A : \text{Set}) \rightarrow A \rightarrow A \\
\text{id}' \ A \ x = \text{id} \ _ \ x
\]

Output:

\[
\text{id} : (A : \text{Set}) \rightarrow A \rightarrow A \\
\text{id} \ A \ x = x
\]

\[
\text{id}' : (A : \text{Set}) \rightarrow A \rightarrow A \\
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Two core computational tasks in elaboration:
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1. $\beta\eta$-conversion checking.
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1. $\beta\eta$-conversion checking.
2. Generating solutions for holes (metavariables).
Solving metas in the standard way

1: Source:

\[ \text{id} : (A : \text{Set}) \to A \to A \]
\[ \text{id} \ A \ x = x \]

\[ \text{id}' : (A : \text{Set}) \to A \to A \]
\[ \text{id}' \ A \ x = \text{id} \ _ \ x \]

2: Plug hole with fresh meta:

\[ \alpha = \lambda A \ x. \ ? \]

3: Solve meta:

\[ \alpha = \lambda A \ x. \ A \]

\[ \text{id} : (A : \text{Set}) \to A \to A \]
\[ \text{id} \ A \ x = x \]

\[ \text{id}' : (A : \text{Set}) \to A \to A \]
\[ \text{id}' \ A \ x = \text{id} \ (\alpha \ A \ x) \ x \]

4: Unfold meta in output:

\[ \text{id} : (A : \text{Set}) \to A \to A \]
\[ \text{id} \ A \ x = x \]

\[ \text{id}' : (A : \text{Set}) \to A \to A \]
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Problems with the standard way

Metas are essentially unscoped: solutions can’t refer to other definitions and meta solutions. Hence: everything must be unfolded.
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Input:

\[\text{id}' : \{A : \text{Set}\} \to A \to A\]
\[\text{id}' = \text{id} \; \text{id} \; \text{id} \; \text{id} \]

Output:

\[\text{id'} : \{A : \text{Set}\} \to A \to A\]
\[\text{id'} = \lambda \{A\} \to\]
\[\quad (\text{id} \; \{(A \to A) \to A \to A\} \to (A \to A) \to A \to A\})\]
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id' : {A : Set} → A → A

id' {A} =

let α : Set = A
    β : Set = α → α
    γ : Set = β → β
    δ : Set = γ → γ

in (id {δ}) (id {γ}) (id {β}) (id {α})
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(Can hash consing help? Not really: overheads and failure to handle beta redexes.)
Scoping for metavariables

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1. Full precision: metas are elaborated into let-definitions in arbitrary local scopes.
   - Dependently typed upgrade of Krishnawami and Dunfield’s mixed-prefix bidirectional checkers.
   - Allows fast let-generalization.
   - More efficient, better output.
   - Challenging to implement.
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2. Limited precision: metas only have top-level scope, and are elaborated into top-level mutual (unordered) definition blocks.
   - Easy to implement.
   - Less efficient and captures less sharing.
   - Implemented in prototype.
Evaluators for elaboration

Recall the two computational tasks: conversion checking, meta solution generation.

Glued values: fully unfolded values, which also carry local values around.

Local values: these are computed to some head normal form while not unfolding some class of definitions.
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Solution: a “glued” evaluator, which computes two different semantic values at the same time.

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2. **Local values**: these are computed to some head normal form while **not** unfolding some class of definitions.
Minimal glued evaluator in Haskell

Glues call-by-need and call-by-name machines together.

data Tm  = Var Int | App Tm Tm | Lam Tm

data Val = VNe Int [Val] [Cl] | VLam [Val] [Cl] Tm

data Cl  = Cl [Cl] Tm

eval :: [Val] → [Cl] → Tm → Val

eval vs cs t = case t of
  Var i → case lookup i vs of
    Just v  → v
    Nothing → VNe (length vs - i - 1) [] []

  App t u → case (eval vs cs t, eval vs cs u) of
    (VLam vs' cs' t', u') → eval (u':vs') (Cl cs u :cs') t'
    (VNe i vs' cs' , u') → VNe i (u':vs') (Cl cs u :cs')

  Lam t → VLam vs cs t
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In principle, one could glue together any number of different evaluators, each optimized for a specific task. Gluing just two machines seems to strike a good balance of complexity and constant overheads.
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We get a larger kernel than in the Coq-style, but benefits seem to be significant.
Prototype implementation

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