`mathlib`: Lean's mathematical library

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Introduction

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  with computable exceptions, e.g. $\mathbb{N}$, $\mathbb{Z}$, lists, ...

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  with computable exceptions, e.g. $\mathbb{N}$, $\mathbb{Z}$, lists, ...
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- Some (current) topics:
  Basic Datatypes, Analysis, Linear Algebra, Set Theory, ...

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  - No general fixpoint operator, no general match operator
    these are derived from recursors

Type classes in Lean

- Type classes are used to fill in implicit values:

$$\texttt{add} : \Pi\{\alpha : \texttt{Type}\}[i : \texttt{has\_add}\ \alpha],\ \alpha \to \alpha \to \alpha$$

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- Output parameters:

$$\begin{aligned}
&\texttt{has_mem} && : \texttt{Type} \to \texttt{out Type} \to \texttt{Type} \\
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- Default values

Library

Library

- Basic (computable) data
- Type class hierarchies:

Orders	orders, lattices
Algebraic	(commutative) groups, rings, fields
Spaces	measurable, topological, uniform, metric

- Set theory (cardinals & ordinals)
- Analysis
- Linear algebra

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- Big operators for `list`, `multiset` and `finset`

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- Existence of inaccessible cardinals (i.e. in the next universe)

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- Infinite sum on topological monoids α:
  $\Sigma : \forall \iota, (\iota \to \alpha) \to \alpha$

Analysis: Analytical Structures as Complete Lattices

Complete lattices, `map` & `comap` as category theory *light*

- Filters, topological spaces, uniform spaces, and measurable spaces form a complete lattices per type

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- Straight forward derivation of continuity rules

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$\texttt{class metric}\ (\alpha : \mathit{Type}) := \ldots$
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Default values give a value for the topology when defining metric

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- Anyway: $\mathbb{R}$ as order & topologically complete field

Linear Algebra

$$\texttt{class}\ \texttt{module}\ (\alpha : \texttt{out}\ \texttt{Type}_u)\ (\beta : \texttt{Type}_v)\ [\texttt{out}\ \texttt{ring}\ \alpha] := \ldots$$

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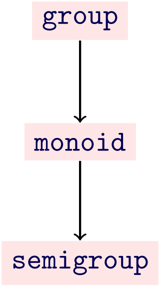
Example

Isomorphism laws:

$$\frac{dom(f)}{ker(f)} \simeq_\ell im(f) \qquad \frac{s}{s \cap t} \simeq_\ell \frac{s \oplus t}{t}$$

Discussion

Problems with Type Classes



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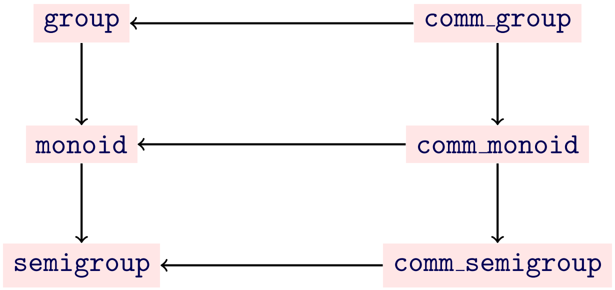

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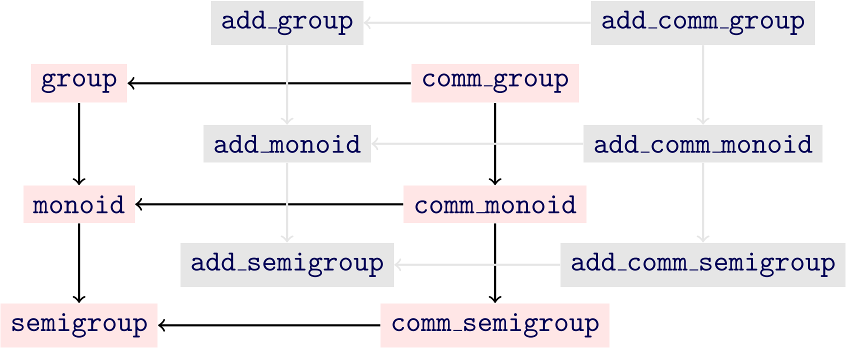

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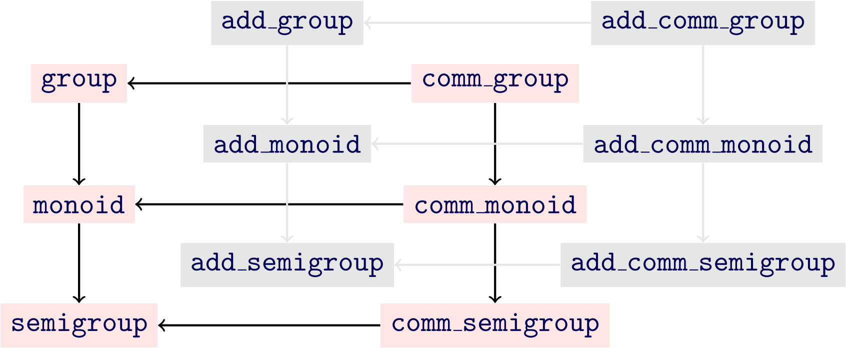

- Currently a automated copy from `group` to `add_group` instead: [*is_group*$(*)(/)(\square^{-1})1$] and [*is_group*$(+)(-)(-\square)0$]
- Mixin type classes
  replace `comm_monoid`, ... by [*is_commutative* $(*)$]

Problem with Universes

```
class functor (M : Type u → Type v) :=
(map : ∀(α β : Type u), (α → β) → M α → M β)
(map_comp : ∀(α β γ : Type u) f g h, map f ∘ map g = map (f ∘ g))
(map_id : ∀α, map id = id)
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$\texttt{class functor}\ (M : \texttt{Type}\ u \to \texttt{Type}\ v) :=$
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\end{aligned}
$$

If we only work with `functor` (`topology` α) our library is too limited, e.g. `topology.map` allows mapping between different universes.

Maintenance

- Currently maintained by Mario Carneiro, me, and Jeremy Avigad
- Contributors:

  Andrew Zipperer, Floris van Doorn, Haitao Zhang, Jeremy Avigad, Johannes Hölzl, Kenny Lau, Kevin Buzzard, Leonardo de Moura, Mario Carneiro, Minchao Wu, Nathaniel Thomas, Parikshit Khanna, Robert Y. Lewis, Simon Hudon
- Currently $\sim$ 51.000 lines of Lean code

mathlib

A (classical) mathematical library for Lean

https://github.com/leanprover/mathlib