CoqHammer: A General-Purpose Automated Reasoning Tool for Coq

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- Automation for Interactive Proof
  - Proof search: intuition, firstorder,
  - Decision Procedures: congruence, fourier, ring, omega, SMTCoq, ...
- AI/ATP techniques: Hammers
  - MizAR for Mizar
  - Sledgehammer for Isabelle/HOL
  - HOL(y)Hammer for HOL Light and HOL4
  - **CoqHammer** for Coq

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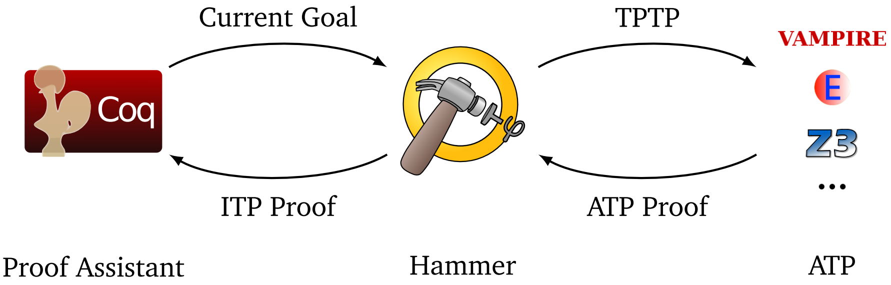
  - We may not know the name of the lemma we want to apply.
  - There may be many equivalent formulations of the lemma – which one is used in the library?
  - The exact lemma may not exist in the library, but it may "trivially" follow from a few other lemmas in the library.

Hammer Overview



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- **Reprove** the conjecture in the logic of the ITP, using the information obtained in the ATP runs. Typically, a list of (usually a few) lemmas needed by an ATP to prove the conjecture is obtained from an ATP run, and we try to reprove the goal from these lemmas.

Evaluations

Top-level goals:

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- **CoqHammer**
  - Coq standard library: 40%

CoqHammer demo

examples/imp.v

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  (to include *all* accessible facts).
- Two machine-learning filters: k-NN and naive Bayes.
- Re-uses the HOLyHammer efficient implementation
  (also adapted by Sledgehammer).

Translation: target logic

Target logic: untyped FOL with equality.

Translation

Three functions $\mathscr{F}$, $\mathscr{G}$, and $\mathscr{C}$.

- $\mathscr{F}$: propositions $\rightarrow$ FOL formulas
  used for CIC_0 terms of type Prop.
- $\mathscr{G}$: types $\rightarrow$ guards
  used for CIC_0 terms of type Type.
- $\mathscr{C}$: all CIC_0 $\rightarrow$ FOL terms

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  For instance, for a (closed) type $\tau = \Pi x : \alpha.\beta(x)$ we have

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  - $\mathscr{C}_\Gamma(\lambda\vec{x} : \vec{t}.s) = F\vec{y}$ where s does not start with a lambda-abstraction any more, F is a fresh constant, $\vec{y} = \mathrm{FV}(\lambda\vec{x} : \vec{t}.s)$ and $\forall\vec{y}.\mathscr{F}_\Gamma(\forall\vec{x} : \vec{t}.F\vec{y}\vec{x} = s)$ is a new axiom.

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- The provers may be run in parallel with different numbers of premises and premise selection methods.

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- 85.2% of proofs reconstructed.

Overall hammer evaluation

All statements from the Coq standard libary

ATP success **50%**

- ATPs used: E, Z3, Vampire with 30 seconds time limit

Overall success **40.8%**

- 8 threads with different lemma selection, premises, provers, reconstruction

Conclusion

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- Improvements needed for dependent types and boolean reflection.

Download

http://cl-informatik.uibk.ac.at/cek/coqhammer/